P-003-1016001

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Seat No. ________

Third Year B. Sc. (Sem. VI) (CBCS) Examination

March / April - 2020

Mathematics : BSMT - 08 (A)

(Graph Theory & Complex Analysis - II)

(New Course)

Faculty Code : 003

Subject Code : 1016001

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All the questions are compulsory.
(2) Figures to the right indicate full marks of the question.

1 (A) Answer the following short answer questions : **4**

(1) Define : Pendant vertex

(2) Define : Unicursal graph

(3) Give number of edges in a Hamiltonian circuit with n vertices.

(4) Define: Rank and nullity of a graph.

(B) Answer in Brief : (Any **One**) **2**

(1) A graph G is a tree then prove that there is exactly one path between every pair of vertices.

(2) Prove that a binary tree with n vertices has $\frac{n+1}{2}$ pendant vertices.

(C) Answer in Detail : (Any **One**) **3**

(1) State and prove necessary and sufficient condition for a graph to be disconnected.

(2) Define radius and diameter of a graph and show by an example that diameter of a graph is not twice the radius of the graph.

(D) Answer in Detail : (Any **One**) **5**

(1) Explain Konigsberg bridge problem.

(2) State and prove a necessary and sufficient condition for a graph to be Euler.

2 (A) Answer the following short answer questions : **4**

(1) Define : Maximal independent set.

(2) Define : Edge connectivity

(3) How many vertices Kuratowski's second graph have ?

(4) Define : Separable graph.

(B) Answer in Brief : (Any **One**) **2**

(1) Define : Planer graph

(2) Define : Proper vertex colouring and chromatic number of a graph.

(C) Answer in Detail : (Any **One**) **3**

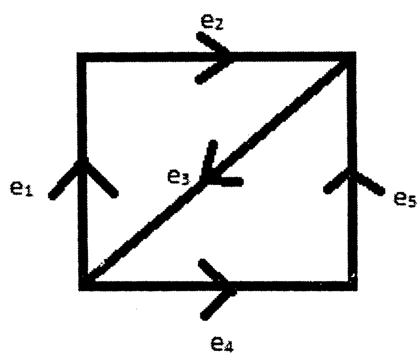
(1) Show that Kuratowski's first graph is non planer.

(2) State the definition and properties of Adjacency matrix.

(D) Answer in Detail : (Any **One**) **5**

(1) If G is a graph with n vertices, e edges, f faces and k components, then show that $n - e + f = k + 1$.

(2) Obtain minimal decyclization of the graph G given below :



3 (A) Answer the following short answer questions : **4**

(1) Define : Bilinear / Mobious transformation.

(2) Define : Inversion Mapping

(3) Define : Rotation Mapping

(4) Define : Fixed points

(B) Answer in Brief : (Any **One**) **2**

(1) Define : Critical points

(2) Find the fixed point transformation for $w = z^2$

(C) Answer in Detail : (Any **One**) **3**

(1) Obtain bilinear transformation which maps the points $z_1 = -i, z_2 = 0, z_3 = i$ of z–plane onto the points of w – plane as $w_1 = -1, w_2 = i$ and $w_3 = 1$.

(2) Prove that $w = \dfrac{az+b}{cz+d}$ is conformal mapping

(D) Answer in Detail : (Any **One**) **5**

(1) Prove that the transformation $(w+1)^2 = \dfrac{4}{z}$ maps the points lying on unit circle in w – plane onto the points lying onto parabola in z – plane.

(2) Show that the set of all bilinear maps for a group under composition of maps.

4 (A) Answer the following short answer questions : **4**

(1) Write the Maclaurin's series expansion of sinz and cosz.

(2) State Taylor's infinite series for analytic function.

(3) Define : Absolute convergence of series.

(4) Write Maclaurin's series for $\dfrac{1}{1+z}$

(B) Answer the following in Brief : (Any **One**) **2**

(1) Obtain Maclaurin's series expansion of coshz.

(2) Find the region of convergence and radius of convergence for $\sum_{1}^{\infty} \dfrac{z^n}{3^n + 1}$

(C) Answer in Detail : (Any **One**) **3**

(1) Expand $\dfrac{1}{(z-1)(z-2)}$ in Laurent's series for $1 < |z| < 2$

(2) State and prove necessary conditions for a series $\sum z_n$ to be convergent.

(D) Answer in Detail : (Any **One**) **5**

(1) State and prove Laurent's theorem for analytic function.

(2) State and prove necessary and sufficient condition for complex sequence $\{Z_n\}$ to be convergent.

5 (A) Answer the following short answer questions : **4**

(1) Define : Isolated singular point.

(2) Define : Residue

(3) Find the zeros of $f(z)=z(z-1)$.

(4) Define : Simple pole

(B) Answer in Brief : (Any **One**) **2**

(1) Find Res $\left(\frac{e^{2z}}{z(z-1)},0\right)$ and Res $\left(\frac{e^{2z}}{z(z-1)},1\right)$

(2) Discuss the method to find residue at simple pole.

(C) Answer in Detail : (Any **One**) **3**

(1) Using Residue theorem, prove that

$$\int_0^\infty \frac{\cos x}{x^2+1}\,dx = \frac{\pi}{2e}$$

(2) Evaluate $\int_{|z|=1} \frac{z\sec z}{(z-1)^2}\,dz$ by using Cauchy' Residue theorem.

(D) Answer in Detail : (Any **One**) **5**

(1) Prove that $\int_0^\infty \frac{dx}{x^4+1} = \frac{\pi}{2\sqrt{2}}$.

(2) Prove : $\int_0^{2\pi} \frac{\cos 3\theta}{5-4\cos\theta}\,d\theta = \frac{\pi}{12}$